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# Scan-Plane Truncation in Diffraction Tomography-Based Imaging

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Three approaches for truncating the scan plane in diffraction tomography-based imaging are suggested and compared. The first and second approaches involve multiplication of a window to the scattered field before evaluating the spatial Fourier transform. The third method relies on an asymptotic end-point expansion of the Fourier integral.

## Introduction

The spatial Fourier transform of the scattered field on a planar scan plane occurs in diffraction tomography-based imaging derived from Devaney's filtered-backpropagation algorithm [1, 2]. To avoid aliasing problems, the scan plane must be large enough to intercept all energy of the scattered field. However, in many practical configurations it is impossible to realize the scan plane size as dictated by this requirement, and a truncated scan plane must therefore be applied. The straight-forward approach is simply to calculate the Fourier transform over the truncated scan plane, corresponding to applying a rectangular window, but this method does not eliminate aliasing. Instead, a window possessing small side lobes in the Fourier domain – as for instance the Blackman-Harris window [3] – is preferred. As an alternative to using windows, the truncation of the scan plane can be accounted for by employing an asymptotic end-point contribution of the Fourier integral.

## The Formulation

The considered measurement configuration is a microwave camera consisting of a transmitting and receiving antenna, operating in fixed offset. The scan is performed with the receiving antenna in the  $z = 0$  plane, and the positions of the receiving antenna and the offset are given by

$$\mathbf{r}_r = x_r \hat{\mathbf{x}} + y_r \hat{\mathbf{y}} \quad \text{and} \quad \mathbf{r}_\Delta = x_\Delta \hat{\mathbf{x}} + y_\Delta \hat{\mathbf{y}} + z_\Delta \hat{\mathbf{z}}, \quad (1)$$

respectively. Both antennas are assumed to be Hertzian dipoles with dipole moments  $\hat{\mathbf{p}}_r$  and  $\hat{\mathbf{p}}_\Delta$  for the receiving and transmitting antennas, respectively. The microwave camera is operating in the frequency range given by the minimum angular frequency  $\omega_{\min}$  and the maximum angular frequency  $\omega_{\max}$ . The scattering objects are positioned in the lower half-space ( $z < 0$ ) and both the scattering objects and the microwave camera are positioned in free space. The non-conducting scattering objects are assumed to have a permittivity  $\Delta\epsilon(\mathbf{r})$  slightly different than that of free space, so that the scattering process can be accurately described by the first-order Born approximation. By application of this approximation, the forward model and imaging algorithm in [4] can be applied, using two identical background media. Using the time factor  $e^{-i\omega t}$ , the imaging algorithm is given by

$$\Delta\epsilon(\mathbf{r}) = \Re \left\{ \frac{64z}{\pi^2 \mu_0^2 c_1^3} \int_{\omega_{\min}}^{\omega_{\max}} \iint_{k_x^2 + k_y^2 < 4k_0^2} \hat{\mathbf{p}}_r \cdot \widetilde{\mathbf{E}}_s(k_x, k_y, \omega) e^{i[k_x(x - \frac{1}{2}x_\Delta) + k_y(y - \frac{1}{2}y_\Delta)]} \right. \\ \left. \cdot \frac{e^{i[z\sqrt{4k_1^2 - k_x^2 - k_y^2} - \frac{1}{2}\Delta z\sqrt{4k_0^2 - k_x^2 - k_y^2}]}{\omega(4k_1^2 - k_x^2 - k_y^2)^{\frac{3}{2}} I(\omega) \hat{\mathbf{p}}_r \cdot \bar{\mathbf{F}}(k_x, k_y, \omega) \cdot [\hat{\mathbf{p}}_\Delta \cdot \bar{\mathbf{F}}(k_x, k_y, \omega)]} dk_y dk_x d\omega \right\}. \quad (2)$$

Herein, the dyadic  $\vec{\vec{F}}$  is the plane-wave expansion of the dyadic Green's function for observation point and source point above and below the  $z = 0$  plane, respectively [4, Eq. 6]. The quantity  $\vec{E}_s$  is the two-dimensional Fourier transform of the scattered field, defined as

$$\vec{E}_s(k_x, k_y, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{E}_s(\mathbf{r}_r, \omega) e^{-i[k_x x_r + k_y y_r]} dx_r dy_r. \quad (3)$$

Since the integration in this expression is to be performed over the entire infinite plane  $z = z_r$ , and the scattered field is only measured in the finite scan plane given by

$$|x_r| \leq x_0 \quad \text{and} \quad |y_r| \leq y_0, \quad (4)$$

only an approximate Fourier transform can be calculated. This will result in a distortion of the calculated image. It is therefore of interest to investigate how the truncation of the scan plane influences the Fourier transform.

### Scan Plane Truncation

Three approaches for truncating the scan plane have been investigated. The first is a simple truncation in which the field outside the scan plane is assumed to be zero. This corresponds to limiting the scan plane by using a simple rectangular window and the resulting expression for the Fourier integral is given as

$$\vec{E}_s^{\text{rect}}(k_x, k_y, \omega) = \int_{-y_0}^{y_0} \int_{-x_0}^{x_0} \mathbf{E}_s(\mathbf{r}_r, \omega) e^{-i[k_x x_r + k_y y_r]} dx_r dy_r. \quad (5)$$

Since the use of the rectangular window is known to give rise to aliasing, the effect of using a more advanced window has also been tested. In this paper the Blackman-Harris window [3] has been applied and the resulting expression for the Fourier transform of the scattered field is

$$\vec{E}_s^{\text{B.H.}}(k_x, k_y, \omega) = \int_{-y_0}^{y_0} \int_{-x_0}^{x_0} w(x_r, x_0) w(y_r, y_0) \mathbf{E}_s(\mathbf{r}_r, \omega) e^{-i[k_x x_r + k_y y_r]} dx_r dy_r \quad (6)$$

where  $w$  is the Blackman-Harris window given by

$$w(x, x_0) = a_0 - a_1 \cos\left(\pi \frac{x + x_0}{x_0}\right) + a_2 \cos\left(2\pi \frac{x + x_0}{x_0}\right) - a_3 \cos\left(3\pi \frac{x + x_0}{x_0}\right) \quad (7)$$

with  $a_0 = 0.35875$ ,  $a_1 = 0.48829$ ,  $a_2 = 0.14128$ , and  $a_3 = 0.01168$ .

The last approach, which has been investigated for compensating for the finite-sized scan plane, is the use of an asymptotic expansion of the semi-infinite part of the Fourier integral in which the scattered field is not measured. By making a far-field approximation of the forward model in [4], the following expression has been found for the scattered field

$$\mathbf{E}_s(\mathbf{r}_r, \omega) = e^{2ik_0 R_c} \mathbf{E}_s^0(\mathbf{r}_r, \omega). \quad (8)$$

Herein,  $k_0$  is the free-space wavenumber and  $R_c$  is the distance from the receiving antenna to the center of the scattering object. The function  $\mathbf{E}_s^0$  is slowly varying compared to the exponential function and the integration over the area outside the scan plane can therefore be evaluated using an asymptotic expansion. Applying this approach, the following expression is found for the Fourier transform of the scattered field

$$\begin{aligned} \widetilde{E_s^{\text{A.E.}}}(k_x, k_y, \omega) = & \int_{-y_0}^{y_0} \int_{-x_0}^{x_0} \mathbf{E}_s(\mathbf{r}_r, \omega) e^{-i[k_x x_r + k_y y_r]} d\mathbf{r}_r \\ & + i \int_0^{2\pi} R_{\min}(\phi) \hat{\mathbf{p}}_r \cdot \mathbf{E}_s(\mathbf{r}_r^{\min}(\phi), \omega) \frac{e^{-iR_{\min}(\phi)[k_x \cos \phi + k_y \sin \phi]}}{2k_1 - (k_x \cos \phi + k_y \sin \phi)} d\phi. \end{aligned} \quad (9)$$

The last integral in (9) is the compensation for the truncated scan plane. The compensation is found as an end-point contribution using partial integration, as described in [5, Ch. 3]. It has been assumed that the scan plane is electrically large and that the center of the scattering object is positioned under the center of the scan plane. The depth at which the scattering object is buried is assumed to be much smaller than the distance from the center to the edge of the scan plane. The distance from the center of the scan plane to the edge is denoted by  $R_{\min}$  and is given as a function of the azimuthal angle  $\phi$  of the receiving antenna. The scan plane with these two parameters is seen in Figure 1. The vector  $\mathbf{r}_r^{\min}$  is the position vector of the receiving antenna when this is positioned at the edge of the scan plane.

Observing the integrand in the compensation integral in (9) it is seen that special care should be taken when applying this algorithm as a singularity is present when

$$2k_1 = k_x \cos \phi + k_y \sin \phi. \quad (10)$$

This singularity is caused by the fact that as the value of  $k_x \cos \phi + k_y \sin \phi$  exceeds  $2k_1$ , the type of asymptotic expansion changes from being an end-point contribution to a stationary-point contribution.

## Numerical Investigations

To investigate the impact of using the three different approaches for truncation, a 2.5-dimensional setup, as the one used in [4], is considered. Herein, an infinitely long cylinder is buried with its axis parallel to the  $x$  axis and the scan is performed along the  $y$  axis with an offset in the  $\hat{y}$  direction of 0.01 m. The cylinder has a radius of  $0.1\lambda$  and is buried with its axis at  $z_c = -2\lambda$ . Both antennas are  $\hat{x}$  directed and the amplitude of the  $\hat{x}$  component of the scattered field  $E_s^x$  as a function of the position of the receiving antenna is seen in Figure 2.

In Figure 3, the amplitudes of the Fourier transforms calculated using the three different truncation techniques are plotted. In each case, the value of  $y_0$  has been chosen to  $2\lambda$ , yielding a total length of the scan line of  $4\lambda$ . In addition to the transforms calculated using truncation, a reference transform found by including the scattered field from  $y_r = -2000\lambda$  to  $y_r = 2000\lambda$  is shown. By comparing the results of the transforms calculated using the truncated signal with the reference transform, it is seen that the transform calculated using the Blackman-Harris window yields the worst results while the transform calculated by adding the asymptotic end-point contribution is the best for the lower values of  $|k_y|$ . For the higher values of  $|k_y|$ , however, the singularity of the asymptotic expansion implies that the accuracy of the transform calculated using a simple rectangular window yields the best results, although it is influenced by aliasing. In Figure 4 the absolute error in phase when comparing the truncated transforms with the reference is plotted. The conclusions on this plot are similar to those of the amplitude plot: The truncation using the Blackman-Harris window yields the worst results while the compensation using the asymptotic expansion yields good results for those values of  $k_y$  for which the singularity of the expansion has no effect on the results.

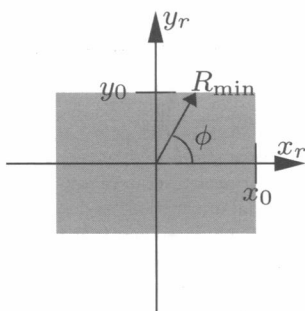


Figure 1: Scan plane and parameters  $R_{min}$  and  $\phi$ .

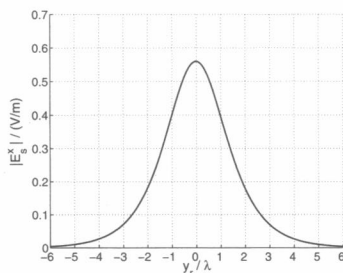


Figure 2: Amplitude of scattered field as function of position of receiving antenna.

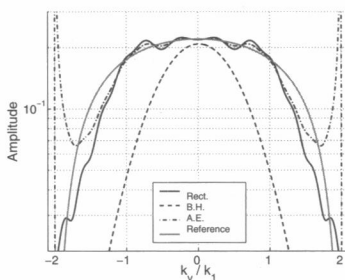


Figure 3: Amplitude of the Fourier transforms calculated using the three different approaches.

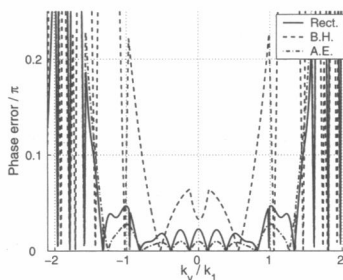


Figure 4: Absolute error in phase of Fourier transforms calculated using the three different approaches.

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